

# Combinatorics

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There's no theory behind combinatorics, you just have to think out of the box.

## 1 Problems

1. In a party with some people, some pairs of them shook hands. Everybody loves the party, so after the party, each person donated \$1 for each other person he shook hands with. Prove that the total amount of dollars donated is even.
2. There are 4 islands and every pair of island is linked with a bridge. Bob wants to travel from island to island in such a way that he crosses each bridge exactly once. He starts from one of the islands. Can he accomplish that? (He can travel to an island more than once. Once he decides to cross a bridge, he must cross all the way to the other side; no U-turning allowed)
3. (Junior 2011) Let  $S_1, S_2, \dots, S_{2011}$  be nonempty sets of consecutive integers such that any 2 of them have a common element. Prove that there is an integer that belongs to every  $S_i, i = 1, \dots, 2011$ . (For example,  $\{2, 3, 4, 5\}$  is a set of consecutive integers while  $\{2, 3, 5\}$  is not.)
4. 25 boys and 25 girls sit around a table. Prove that it is always possible to find a person both of whose neighbours are girls.
5. (Junior 2015) Let  $30x0\bar{7}0y03$  be a 9-digit integer. Find all possible value of the pair  $(x, y)$ , so that  $n$  is a multiple of 37.
6. (Junior 2013) Let  $a < b < c < d < e$  be real numbers. Among the 10 sums of the pairs of these numbers, the least three are 32, 36 and 37 while the largest two are 48 and 51. Find all possible values of  $e$ .
7. (Junior 2013) Six musicians gathered at a chamber music festival. At each scheduled concert some of the musicians played while the others listened as members of the audience. What is the least number of such concerts which would need to be scheduled so that for every two musicians each must play for the other in some concert.
8. (Junior 2016) A group of tourists get on 10 buses in the outgoing trip. The same group of tourists get on 8 buses in the return trip. Assuming each bus carries at least 1 tourist, prove that there are at least 3 tourists such that each of them has taken a bus in the return trip that has more people than the bus he has taken in the outgoing trip.