Multiple Choice Questions

1. Let b be a positive integer. If the minimum possible value of the quadratic function $5x^2 + bx + 506$ is 6, find the value of b.

   (A) 90 (B) 100 (C) 110 (D) 120 (E) 130

2. Which of the following is equal to

$$\sqrt{5+\sqrt{3}} + \sqrt{5-\sqrt{3}}\ ?$$

   (A) $\sqrt{10-\sqrt{22}}$ (B) $\sqrt{10+\sqrt{22}}$ (C) $\sqrt{10-2\sqrt{22}}$
   (D) $\sqrt{10+2\sqrt{22}}$ (E) None of the above

3. Simplify

$$\log_8 5 \cdot (\log_5 3 + \log_{25} 9 + \log_{125} 27).$$

   (A) $\log_2 3$ (B) $\log_3 2$ (C) $\log_2 9$ (D) $\log_3 16$ (E) $\log_2 27$

4. Let $a = 50^{\frac{1}{505}}$, $b = 10^{\frac{1}{303}}$ and $c = 6^{\frac{1}{202}}$. Which of the following is true?

   (A) $a < b < c$ (B) $a < c < b$ (C) $b < a < c$ (D) $b < c < a$ (E) $c < b < a$

5. Let $p = \log_{10}(\sin x)$, $q = (\sin x)^{10}$, $r = 10^{\sin x}$, where $0 < x < \frac{\pi}{2}$. Which of the following is true?

   (A) $p < q < r$ (B) $p < r < q$ (C) $q < r < p$ (D) $q < p < r$ (E) $r < p < q$

Short Questions

6. Find the minimum possible value of $|x - 10| - |x - 20| + |x - 30|$, where x is any real number.

7. Parallelogram $ABCD$ has sides $AB = 39$ cm and $BC = 25$ cm. Find the length of diagonal AC (in cm) if diagonal $BD = 34$ cm.

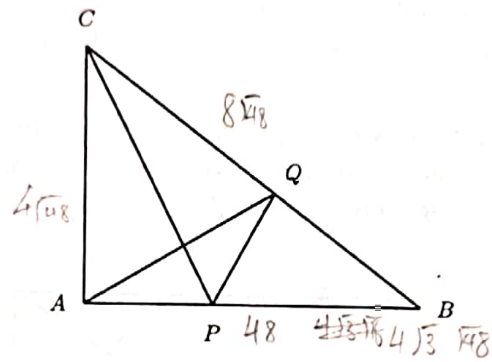
8. Suppose $\sin(45° - x) = -\frac{1}{3}$, where $45° < x < 90°$. Find $(6\sin x - \sqrt{2})^2$.

9. If $8\cos x - 8\sin x = 3$, find the value of $55\tan x + \dfrac{55}{\tan x}$.

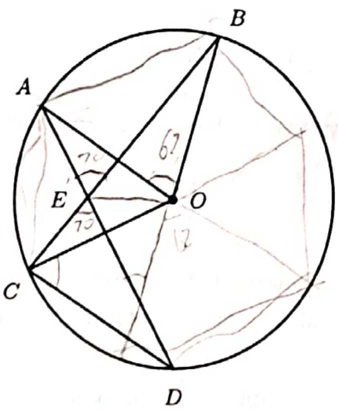
10. Find the number of ordered pairs (x, y), where x and y are integers, such that

$$x^2 + y^2 - 20x - 14y + 140 < 0.$$

11. The figure below shows a right-angled triangle ABC such that $\angle BAC = 90^\circ$, $\angle ABC = 30^\circ$ and $AB = 48$ cm. Let P be a point on side AB such that CP is the angle bisector of $\angle ACB$ and Q be a point on side BC such that line AQ is perpendicular to line CP. Determine the length of PQ.



12. In the figure below, the point O is the center of the circle, AD and BC intersect at E, and $\angle AEB = 70^\circ$, $\angle AOB = 62^\circ$. Find the angle $\angle OCD$ (in degree °).



13. Find the value of $\dfrac{4\cos 43^\circ}{\sin 73^\circ} - \dfrac{12\sin 43^\circ}{\sqrt{3}\sin 253^\circ}$.

14. If $\dfrac{x^2}{5} + \dfrac{y^2}{7} = 1$, find the largest possible value of $(x+y)^2$.

15. Find the coefficient of x^6 in the expansion of $(1 + x + 2x^2)^7$.

16. Suppose $(3x-y)^2 + \sqrt{x + 38 + 14\sqrt{x-11}} + |z + x - y| = 7$. Find the value of $|x+y+z|$.

17. Suppose there are real numbers x, y, z satisfying the following equations:

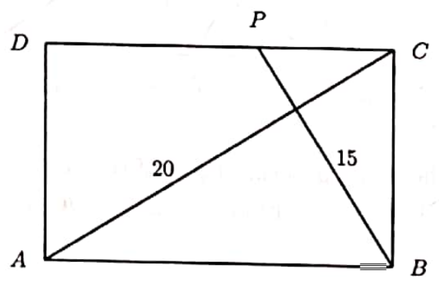
$$x + y + z = 60, \quad xy - z^2 = 900.$$

Find the maximum possible value of $|z|$.

18. Find the sum $\displaystyle\sum_{k=1}^{16} \log_2 \left(\sqrt{\sin^2 \frac{k\pi}{8} + 1} - \sin \frac{k\pi}{8} \right)$

19. Let a, b be positive real numbers, where $a > b$. Suppose there exists a real number x such that $(\log_2 ax)(\log_2 bx) + 25 = 0$. Find the minimum possible value of $\frac{a}{b}$.

20. The figure below shows a rectangle $ABCD$ such that the diagonal $AC = 20$ cm. Let P be a point on side CD such that BP is perpendicular to diagonal AC. Find the area of rectangle $ABCD$ (in cm^2) if $BP = 15$ cm.



21. Find the smallest positive integer that is greater than the following expression:

$$\left(\sqrt{7} + \sqrt{5}\right)^4.$$

22. Find the number of non-congruent right-angled triangles such that the length of all their sides are integers and that the hypotenuse has a length of 65 cm.

23. There are 6 couples, each comprising a husband and a wife. Find the number of ways to divide the 6 couples into 3 teams such that each team has exactly 4 members, and that the husband and the wife from the same couple are in different teams.

24. The **digit sum** of a number, say 987, is the sum of its digits, $9 + 8 + 7 = 24$. Let A be the digit sum of 2020^{2021}, and let B be the digit sum of A. Find the digit sum of B.

25. $40 = 2 \times 2 \times 2 \times 5$ is a positive divisor of 1440 that is a product of 4 prime numbers. $48 = 2 \times 2 \times 2 \times 2 \times 3$ is a positive divisor of 1440 that is a product of 5 prime numbers. Find the sum of all the positive divisors of 1440 that are products of an odd number of prime numbers.