Multiple Choice Questions

1. If α, β are the roots of the equation $3x^2 + x - 1 = 0$, where $\alpha > \beta$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

   (A) $\frac{7}{9}$ (B) $-\frac{7}{9}$ (C) $\frac{7}{3}$ (D) $-\frac{7}{3}$ (E) $-\frac{1}{9}$

2. Find the value of

$$\frac{2014^3 - 2013^3 - 1}{2013 \times 2014}.$$

   (A) 3 (B) 5 (C) 7 (D) 9 (E) 11

3. Find the value of $\dfrac{\log_5 9 \log_7 5 \log_3 7}{\log_2 \sqrt{6}} + \dfrac{1}{\log_9 \sqrt{6}}$.

   (A) 2 (B) 3 (C) 4 (D) 6 (E) 7

4. Find the smallest number among the following numbers:

   (A) $\sqrt{55} - \sqrt{52}$ (B) $\sqrt{56} - \sqrt{53}$ (C) $\sqrt{77} - \sqrt{74}$
   (D) $\sqrt{88} - \sqrt{85}$ (E) $\sqrt{70} - \sqrt{67}$

5. Find the largest number among the following numbers:

   (A) 30^{30} (B) 50^{10} (C) 40^{20} (D) 45^{15} (E) 5^{60}

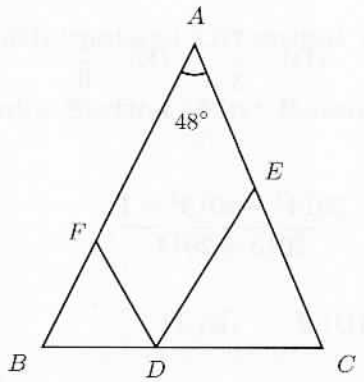
6. Given that $\tan A = \frac{12}{5}$, $\cos B = -\frac{3}{5}$ and that A and B are in the same quadrant, find the value of $\cos(A - B)$.

   (A) $-\frac{63}{65}$ (B) $-\frac{64}{65}$ (C) $\frac{63}{65}$ (D) $\frac{64}{65}$ (E) $\frac{65}{63}$

7. Find the largest number among the following numbers:

   (A) $\tan 47^\circ + \cos 47^\circ$ (B) $\cot 47^\circ + \sqrt{2} \sin 47^\circ$ (C) $\sqrt{2} \cos 47^\circ + \sin 47^\circ$
   (D) $\tan 47^\circ + \cot 47^\circ$ (E) $\cos 47^\circ + \sqrt{2} \sin 47^\circ$

8. In the diagram below, $\triangle ABC$ is a triangle and D, E, F are points on BC, CA, AB respectively. It is given that $BF = BD$, $CD = CE$ and $\angle BAC = 48°$. Find the angle $\angle EDF$.



(A) 64° (B) 66° (C) 68° (D) 70° (E) 72°

9. Find the number of real numbers which satisfy the equation

$$x|x-1| - 4|x| + 3 = 0.$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

10. If $f(x) = \dfrac{1}{x} - \dfrac{4}{\sqrt{x}} + 3$ where $\dfrac{1}{16} \le x \le 1$, find the range of $f(x)$.

(A) $-2 \le f(x) \le 4$ (B) $-1 \le f(x) \le 3$ (C) $0 \le f(x) \le 3$
(D) $-1 \le f(x) \le 4$ (E) None of the above

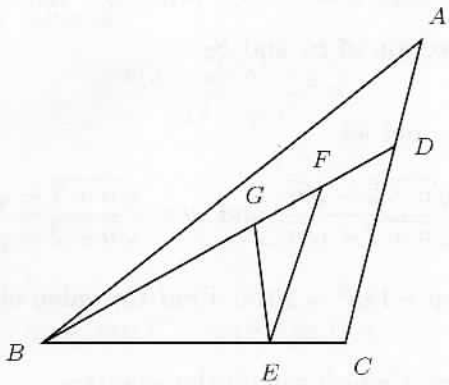
Short Questions

11. Suppose x is real number such that $\dfrac{27 \cdot 9^x}{4^x} = \dfrac{3^x}{8^x}$. Find the value of $2^{-(1+\log_2 3)x}$.

12. Evaluate $50\,(\cos 39° \cos 21° + \cos 129° \cos 69°)$.

13. Suppose a and b are real numbers such that the polynomial $x^3 + ax^2 + bx + 15$ has a factor of $x^2 - 2$. Find the value of a^2b^2.

14. In the triangle $\triangle ABC$ below, $AC = 3AD$, $BC = 4EC$, $BD = 5GF = 5FD$. Suppose the area of $\triangle ABC$ is 900 meter^2. Find the area of the triangle $\triangle EFG$ in meter^2.



15. Let x, y be real numbers such that $y = |x - 1|$. What is the smallest value of $(x - 1)^2 + (y - 2)^2$?

16. Evaluate the sum

$$\frac{3! + 4!}{2(1! + 2!)} + \frac{4! + 5!}{3(2! + 3!)} + \cdots + \frac{12! + 13!}{11(10! + 11!)}.$$

17. Let n be a positive integer such that $12n^2 + 12n + 11$ is a 4-digit number with all 4 digits equal. Determine the value of n.

18. Given that in the expansion of $(2 + 3x)^n$, the coefficients of x^3 and x^4 are in the ratio $8 : 15$. Find the value of n.

19. In a triangle $\triangle ABC$, it is given that

$$(\sin A + \sin B) : (\sin B + \sin C) : (\sin C + \sin A) = 9 : 10 : 11.$$

Find the value of $480 \cos A$.

20. Let $x = \sqrt{37 - 20\sqrt{3}}$. Find the value of

$$\frac{x^4 - 9x^3 + 5x^2 - 7x + 68}{x^2 - 10x + 19}.$$

21. Let n be an integer, and let $\triangle ABC$ be a right-angled triangle with right angle at C. It is given that $\sin A$ and $\sin B$ are the roots of the quadratic equation

$$(5n + 8)x^2 - (7n - 20)x + 120 = 0.$$

Find the value of n.

22. Let S_1 and S_2 be sets of points on the coordinate plane $\mathbb{R}^2$ defined as follows:

$$S_1 = \{(x, y) \in \mathbb{R}^2 \ : \ |x + |x|| + |y + |y|| \leq 2\},$$

$$S_2 = \{(x, y) \in \mathbb{R}^2 \ : \ |x - |x|| + |y - |y|| \leq 2\}.$$

Find the area of the intersection of S_1 and S_2.

23. Let n be a positive integer, and let

$$x = \frac{\sqrt{n+2} - \sqrt{n}}{\sqrt{n+2} + \sqrt{n}} \text{ and } y = \frac{\sqrt{n+2} + \sqrt{n}}{\sqrt{n+2} - \sqrt{n}}.$$

It is given that $14x^2 + 26xy + 14y^2 = 2014$. Find the value of n.

24. Find the number of integers x which satisfy the equation

$$(x^2 - 5x + 5)^{x+5} = 1.$$

25. Find the number of ordered pairs of integers (p, q) satisfying the equation

$$p^2 - q^2 + p + q = 2014.$$

26. Suppose x is measured in radians. Find the maximum value of

$$\frac{\sin 2x + \sin 4x + \sin 6x}{\cos 2x + \cos 4x + \cos 6x}$$

for $0 \leq x \leq \frac{\pi}{16}$.

27. Determine the number of ways of colouring a 10×10 square board using two colours black and white such that each 2×2 subsquare contains 2 black squares and 2 white squares.

28. In the isosceles triangle ABC with $AB = AC$, D and E are points on AB and AC respectively such that $AD = CE$, and $DE = BC$. Suppose $\angle AED = 18°$. Find the size of $\angle BDE$ in degree.

29. Find the number of ordered triples of real numbers (x, y, z) that satisfy the following system of equations:

$$\begin{aligned} x^2 &= 4y - 4, \\ y^2 &= 4z - 4, \\ z^2 &= 4x - 4. \end{aligned}$$

30. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4\}$. Find the number of 4-element subsets Y of X such that $10 \in Y$ and the intersection of Y and A is not empty.

31. Find the number of ways that 7 different guests can be seated at a round table with exactly 10 seats, without removing any empty seats. Here, two seatings are considered to be the same if they can be obtained from each other by a rotation.

32. Determine the maximum value of

$$\frac{8(x+y)(x^3+y^3)}{(x^2+y^2)^2}$$

for all $(x, y) \neq (0, 0)$.

33. Find the value of

$$2\left(\sin 2^\circ \tan 1^\circ + \sin 4^\circ \tan 1^\circ + \sin 6^\circ \tan 1^\circ + \cdots + \sin 178^\circ \tan 1^\circ\right).$$

34. Let $x_1, x_2, \ldots, x_{100}$ be real numbers such that

$$|x_1| = 63, \quad \text{and} \quad |x_{n+1}| = |x_n + 1| \quad \text{for } n = 1, 2, \ldots, 99.$$

Find the largest possible value of $(-x_1 - x_2 - \cdots - x_{100})$.

35. As shown in the figure below, two circles ω, ω', with centers O and O' respectively, intersect at the points C and D. The straight lines CD and $BYXA$ intersect at the point Z. Moreover, the straight line WB is tangent to both of the circles. Suppose $ZX = ZY$ and $AB \cdot AX = 100$. Find the value of BW.