Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2018

(Open Section, Round 2)

Saturday, 30 June 2018 **0900-1300**

INSTRUCTIONS TO CONTESTANTS

1. *Answer ALL 5 questions.*
2. *Show all the steps in your working.*
3. *Each question carries 10 marks.*
4. *No calculators are allowed.*

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1. Consider a regular cube with side length 2. Let A and B be two vertices that are furthest apart. Construct a sequence of points on the surface of the cube $A_1, A_2, \ldots, A_k$ so that $A_1 = A$, $A_k = B$ and for any $i = 1, \ldots, k-1$, the distance from A_i to A_{i+1} is 3. Find the minimum value of k.

2. Let O be a point inside a triangle ABC such that $\angle BOC = 90°$ and $\angle BAO = \angle BCO$. Prove that $\angle OMN = 90°$, where M, N are the midpoints of AC and BC respectively.

3. Let n be a positive integer. Show that there exists an integer m such that

$$2018m^2 + 20182017m + 2017$$

is divisible by 2^n.

4. Each of the squares in a 2×2018 grid of squares is to be coloured black or white such that in any 2×2 block, at least one of the 4 squares is white. Let P be the number of ways colouring the grid. Find the largest k so that 3^k divides P.

5. Consider a polynomial $P(x, y, z)$ in three variables with integer coefficients such that for any real numbers a, b, c,

$$P(a, b, c) = 0 \Leftrightarrow a = b = c.$$

Find the largest integer r such that for all such polynomials $P(x, y, z)$ and integers n, m,

$$m^r \mid P(n, n+m, n+2m).$$