Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2016

Junior Section (Round 1)

Tuesday, 31 May 2016

0930-1200 hrs

Instructions to contestants

1. *Answer ALL 35 questions.*
2. *Enter your answers on the answer sheet provided.*
3. *For the multiple choice questions, enter your answer on the answer sheet by shading the bubble containing the letter (A, B, C, D or E) corresponding to the correct answer.*
4. *For the other short questions, write your answer on the answer sheet and shade the appropriate bubble below your answer.*
5. *No steps are needed to justify your answers.*
6. *Each question carries 1 mark.*
7. *No calculators are allowed.*
8. *Throughout this paper, let* $\lfloor x \rfloor$ *denote the greatest integer less than or equal to* x. *For example,* $\lfloor 2.1 \rfloor = 2$, $\lfloor 3.9 \rfloor = 3$.
9. *Throughout this paper, let* $[A_1A_2\ldots A_r]$ *denote the area of the polygon* $A_1A_2\ldots A_r$.
10. *Throughout this paper, let* $\overline{a_{n-1}a_{n-2}\ldots a_0}$ *denote an* n*-digit number with the digits* a_i *in the corresponding position, i.e.* $\overline{a_{n-1}a_{n-2}\ldots a_0} = a_{n-1}10^{n-1} + a_{n-2}10^{n-2} + \cdots + a_0 10^0$.

PLEASE DO NOT TURN OVER UNTIL YOU ARE TOLD TO DO SO

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Multiple Choice Questions

1. We know that I^2=ME and I^3=YOU. If each distinct letter represents a unique digit such that the arithmetic holds, what is the value of E?

   (A) 4 (B) 5 (C) 6 (D) 8 (E) 9

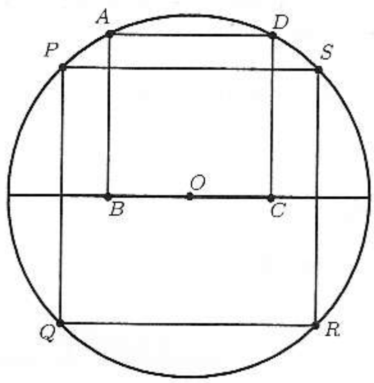
2. Given three integers, we form another three integers by adding the mean of any two of them to the third. If the three new integers formed are 33, 35 and 40, find the mean of the original three integers.

   (A) 18 (B) 27 (C) 30 (D) 36 (E) None of the above

3. Let $ABCD$ be a rectangle, E be a point on BC such that $BE = 2EC$, and F be a point on AE such that $AF = 3FE$. If the area of $ABCD$ is 1200, what is the area of the quadrilateral $DFEC$?

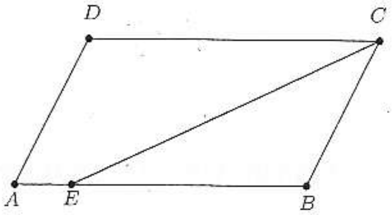
   (A) 100 (B) 300 (C) 350 (D) 400 (E) None of the above

4. The diagram shows a square $ABCD$ inscribed in a semicircle with centre O and another square $PQRS$ inscribed in the entire circle with the same centre. If the area of $ABCD$ is 16, find the area of $PQRS$.



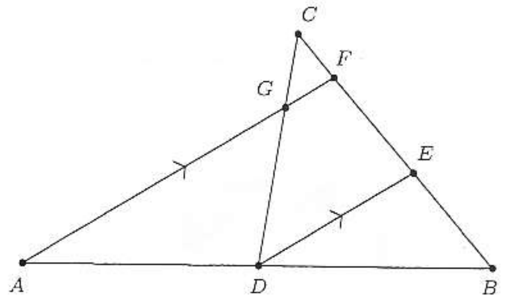
   (A) 20 (B) 24 (C) 32 (D) 40 (E) 48

5. $ABCD$ is a parallelogram. E is a point on the side AB such that the ratio of the area of the quadrilateral $AECD$ to the area of the triangle ABC is $7 : 5$. The ratio of $AE : EB$ is



   (A) 2:3 (B) 2:7 (C) 3:4 (D) 3:5 (E) 5:7

6. In the diagram D is the midpoint of AB, $EF = 2CF$ and AF is parallel to DE. Given the area of triangle GFC is 4, find the area of triangle ADG.



(A) 32 (B) 36 (C) 40 (D) 64 (E) 76

7. Which of the following five numbers has the greatest value?

(A) $(10-\pi)(10+\pi)$ (B) $(11-\pi)(9+\pi)$ (C) $(12-\pi)(8+\pi)$
(D) $(13-\pi)(7+\pi)$ (E) $(14-\pi)(6+\pi)$

8. Two real numbers u and v satisfy the following equations respectively:

$$2015u^2 + 2016u + 1 = 0;$$
$$v^2 + 2016v + 2015 = 0.$$

If $uv \neq 1$, find the value of $\dfrac{u}{v}$

(A) $-\dfrac{2016}{2015}$ (B) $\dfrac{1}{2015}$ (C) $\dfrac{1}{2016}$ (D) $\dfrac{2016}{2015}$ (E) 1

9. Find the minimum value of the real valued function $x + 2016 - \sqrt{2x-3}$.

(A) 2015 (B) 2016 (C) 2017 (D) 2018 (E) 2019

10. The number 123456789101112... is formed by writing the whole numbers $1, 2, 3, \ldots$ until there are 201 digits in the number. Find the remainder when this number is divided by 9.

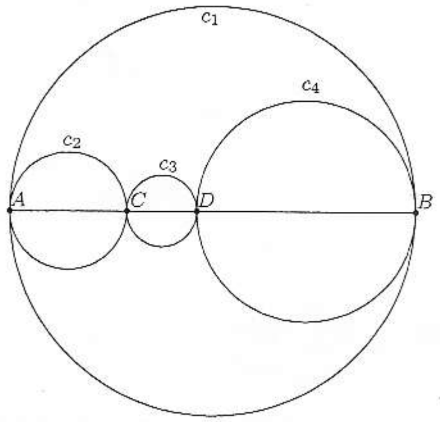
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Short Questions

11. If the sum and product of two positive real numbers are both equal to 13, find the sum of the squares of these two numbers.

12. Let x be a positive integer. If the highest common factor of x and 168 is 12 and the highest common factor of x and 270 is 18, find the smallest possible value of x.

13. In the diagram, AB, AC, CD and DB are respectively the diameters of the circles c_1, c_2, c_3 and c_4. If the circumference of c_1 is 2016, what is the sum of the circumferences of all four circles?

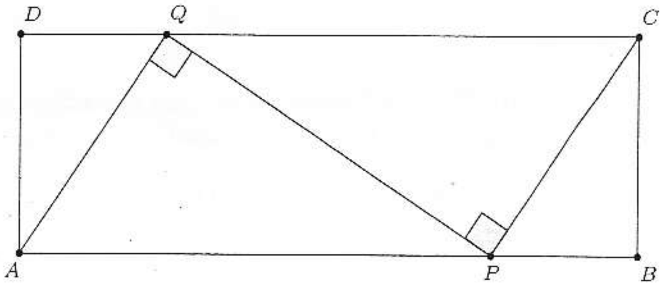


14. Find the integer closest to $10\sqrt{0.7} + 10\sqrt{2.8}$.

15. In the figure below, each distinct letter represents a unique digit such that the arithmetic sum holds. What is the five digit number represented by SIXTY?

	F	O	R	T	Y
			T	E	N
+			T	E	N
	S	I	X	T	Y.

16. In the diagram, $ABCD$ is a rectangle in which $AB = 34$ and $BC = 12$. P and Q are points on AB and CD respectively such that $\angle CPQ$ and $\angle PQA$ are right angles. Find the sum of the two possible lengths of side AP.



17. Let a, b and c be positive integers such that

$$ab + ac = 144$$
$$ab + bc = 209$$
$$ac + bc = 221.$$

Find the value of $a^2 + b^2 + c^2$.

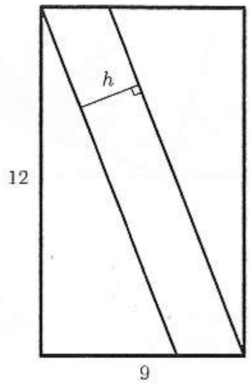
18. P and Q are two regular polygons with respectively n and m number of sides. The ratio of the interior angles of P and Q is $4 : 3$. If $n > m$, how many possible pairs of (n, m) are there?

19. It is given that x and y are positive integers such that $x > y$ and

$$\sqrt{x} + \sqrt{y} = \sqrt{2000}.$$

How many different possible values can x can take?

20. The diagram shows a 12 by 9 rectangle which is cut by a pair of parallel line segments into three parts with the equal areas. If h denotes the distance between the two parallel lines, find the value of $30h^2$.



21. An examination comprises two papers each with a total of 100 marks. In order to pass the examination, a candidate must score at least 45 marks in each paper and at least 100 marks on the two papers combined. Only integer marks will be given for each paper. Find the number of possible ways in which a candidate scores at least 45 marks in each paper and yet fails the examination.

22. Find the sum of all the possible three digit numbers $\overline{abc}$ such that the six digit number $\overline{741abc}$ is divisible by 6, 7 and 10.

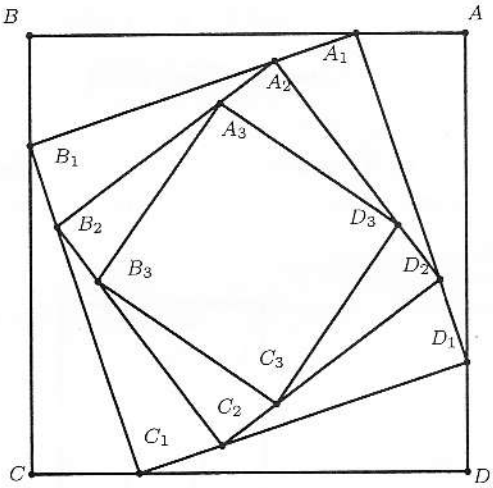
23. A and B are two right circular cylinders. The curved surface area of A is 12.5% more than that of B while the base area of A is 19% less than that of B. If the height of A is x% more than that of B, find the value of x.

24. If $\dfrac{a}{2b} = \dfrac{2b}{3c} = \dfrac{3c}{8a}$, find the value of $\dfrac{ac + cb}{cb - ba}$.

25. Find the number of pairs of positive integers x and y which satisfy the equation

$$3x + 5y = 2016.$$

26. The diagram shows a series of inscribed squares. The area of the largest outer square $ABCD$ is 512. The first inner square is $A_1B_1C_1D_1$ where $AA_1 = \frac{1}{4}AB$, $BB_1 = \frac{1}{4}BC$, $CC_1 = \frac{1}{4}CD$ and $DD_1 = \frac{1}{4}DA$. The second inner square is $A_2B_2C_2D_2$ where $A_1A_2 = \frac{1}{4}A_1B_1$, $B_1B_2 = \frac{1}{4}B_1C_1$ and so on. The third inner square which is the smallest in the diagram is formed in a similar way. Find the area of the smallest square.



27. Let $n_1, n_2, n_3, \ldots, n_9$ be nine distinct positive integers such that $n_1 < n_2 < n_3 < \ldots < n_9$ and $n_1 + n_2 + n_3 + \ldots + n_9 = 180$. Suppose that the value of $n_1 + n_2 + n_3 + n_4 + n_5$ is maximum, find the maximum possible value of $n_9 - n_1$.

28. Let x, y and z be positive integers that satisfy the equations

$$x^2 + y - z = 124 \qquad \text{and} \qquad x + y^2 - z = 100.$$

Find the value of $x + y + z$.

29. Find the positive integer n such that

$$\frac{1}{n^2+5n+6} + \frac{1}{n^2+7n+12} + \frac{1}{n^2+9n+20} = \frac{1}{270}.$$

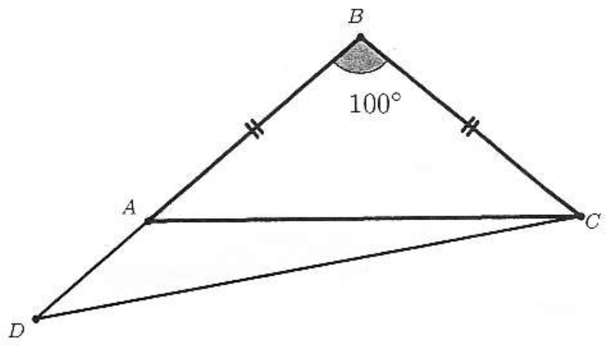
30. ABC is an isosceles triangle with $AB = AC = 3$. There are k distinct points on BC, denoted by $P_1, P_2, \ldots, P_k$. Let $x_i = AP_i^2 + BP_i \cdot P_iC$ where $i = 1, 2, \ldots, k$. Find the value of k if $x_1 + x_2 + \ldots + x_k = 1080$.

31. If a and b are integers and $\sqrt{3 - 2\sqrt{2}}$ is one of the roots of the equation $x^2 + ax + b = 0$, find the value of $a - b$.

32. Find the smallest integer n such that

$$2n \cdot \left(45 - \sqrt{2016}\right) > 1.$$

33. The diagram shows an isosceles triangle ABC with $BA = BC$ and $\angle ABC = 100°$. If D is a point on BA produced such that $BD = AC$, find $\angle BDC$.



34. Let x, y and z be positive real numbers such that $x + y + z = 1$ and $xy + yz + zx = \frac{1}{3}$. Find the value of

$$\frac{4x}{y+1} + \frac{16y}{z+1} + \frac{64z}{x+1}.$$

35. Find the sum of all positive integers n such that $n^2 + n + 2016$ is a perfect square.